2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Ensuring Network Connectivity for Nonholonomic Robots During Rendezvous

†Z. Kan, †A. P. Dani, ‡J. M. Shea, and †,‡W. E. Dixon
†Department of Mechanical and Aerospace Engineering,
‡Department of Electrical and Computer Engineering,
University of Florida, Gainesville, FL 32611-6250, USA.
Email: kanzhen0322@ufl.edu, ashwin31@ufl.edu, jshea@ece.ufl.edu, wdixon@ufl.edu

Abstract— **To achieve a cooperative objective in a multi-robot system typically requires that the robots collaborate over a communication network. In this paper, we design a control strategy for repositioning and reorienting a group of wheeled robots with nonholonomic constraints and limited communication and sensing capabilities. Each robot knows the positions of only those nodes within its sensing range and can only communicate with nodes within its communication range. Thus, the objective must be accomplished while ensuring that specified nodes stay within each other's sensing and communication ranges and that the overall communication network stays connected. To achieve these objectives, we develop a dipolar navigation function and corresponding time-varying continuous controller. We show that if the network is initially connected, the controller maintains the specified communication links at all times while moving the robots into the specified positions and orientations. We consider the particular application of moving the robots to a common rendezvous point with a specified orientation. Simulation results verify the effectiveness of the proposed approach.**

I. Introduction

A wide range of applications can benefit or potentially require collaborative motion of a multi-robot system, e.g., foraging, surveillance, search, rescue and mobile target tracking. In general, the domain of multi-robot systems can be divided into two types according to whether or not cooperation among robots is required to achieve a task. Contrary to a non-cooperative multi-robot system, where each agent makes an independent decision based on its own states, a networked multi-robot system requires robots to perform the task by taking into account the states of other robots. Communication and the mutual exchange of information among the team members are key performance factors for such cooperative multi-robot systems; however, difficulties arising from limited communication and sensing capabilities on each robot can cause the network to partition. When the network partitions, communication between groups of robots can be permanently severed, leading to a mission failure.

Motivated by the practical need to maintain network connectivity, recent results have been developed to ensure connectivity in flocking control [1], [2], formation control [3], rendezvous [4]–[7] and other applications [8]–[10]. In most of the aforementioned work, only linear models of motion are taken into account, i.e., the first order integrator. In this paper, a group of wheeled mobile robots with nonholonomic constraints are considered. Each robot is assumed to be equipped with a passive range sensor (e.g., a camera), to provide local feedback of the relative trajectory of other robots within a limited sensing region, and some form of transceiver that can be used to broadcast information to immediate neighbors. The objective is to steer the multi-robot system to a common setpoint with a desired orientation, while maintaining network connectivity during the evolution.

Due to the Brockett's necessary smooth feedback stabilization condition [11], a nonholonomic system can only be stabilized to an equilibrium point using either a time-varying continuous or time-invariant discontinuous state feedback control law. Numerous results have been developed for the stabilization of a single robot with nonholonomic constraints in the past decades. However, such controllers may not be applicable for a networked multi-robot system with a cooperative objective, e.g., maintaining network connectivity. Navigation functions, a particular class of potential functions, provide an alternative for the navigation of either a single mobile robot or a multi-robot system. The navigation function developed in [12] and [13] is a real-valued function that is designed so that the negated gradient field does not have a local minima. As such, closed-loop navigation function techniques guarantee convergence to a desired destination. To facilitate the control design for the nonholonomic navigation to the destination with desired orientation, a dipolar navigation function was proposed and a discontinuous time-invariant controller was developed to navigate a single robot in [14]. The work in [14] was then extended to a multi-robot system with both holonomic and nonholonomic robots in [15] and extended to navigate a nonholonomic system in three dimensions case in [16]. However, only a time-invariant discontinuous controller was developed in [14]–[16] when combined with the navigation function framework. A navigation function framework is also used in [15], [17]–[20], where agents acted independently and were not required to achieve a network objective, while results in [21]–[23] use potential fields/navigation functions to achieve a cooperative network objective (e.g., formation control or consensus) under the assumption that the agents

[1]This research is supported by National Science Foundation grant number CNS-0626863.

can always communicate (i.e., the graph nodes are assumed to remain connected). When considering maintenance of the network connectivity, a discontinuous controller, similar to [14], was used to steer a multi-robot system with nonholonomic constraints to rendezvous at a common position in [24]. However, each robot can only achieve the destination with arbitrary orientation and has to reorient at the destination. Moreover, the multi-robot system can not converge to any arbitrary destination, and their destination only depends on the initial deployment in [24].

In contrast, a continuous time-varying controller based on a dipolar navigation function is developed in this paper to steer a group of wheeled mobile robots to a specified common setpoint with a desired orientation, while also achieving a cooperative objective of maintaining the network connectivity. Based on our previous work in [3], the dipolar navigation function structure in [14] is modified to stabilize the nonholonomic system and maintain connectivity of the network. Distinguishing factors of this work include: 1) limited communication and sensing capabilities are taken into account in the control design, and the system is guaranteed to maintain connectivity during the evolution if the initial system is connected; 2) a time-varying continuous controller is developed to steer the robots to a common destination with a desired orientation; 3) by using the navigation function framework, each robot is guaranteed to be steered and stabilized at the destination without being trapped by local minima; 4) our result is not restricted to the rendezvous problem, and it can be easily extended to other applications by replacing the objective function in the navigation function to accommodate different tasks, such as formation control, flocking, and the consensus problem.

II. Problem Formulation

Consider a network composed of N Wheeled Mobile Robots (WMR) operating in the workspace $\mathcal{F} \in \mathbb{R}^2$, where robot i moves according to the following nonholonomic kinematics:

$$\dot{q}_i = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} u_i, \; i = 1, \cdots, N \tag{1}$$

where $q_i(t)$ denotes the states of robot i, defined as $q_i(t) \triangleq \begin{bmatrix} p_i^T(t) & \theta_i(t) \end{bmatrix}^T \in \mathbb{R}^3$ with $p_i \triangleq \begin{bmatrix} x_i(t) & y_i(t) \end{bmatrix}^T \in \mathbb{R}^2$ denoting the position of robot i, and $\theta_i \in (-\pi, \pi]$ denoting its orientation with respect to the global coordinate frame in $\mathcal{F}$. In (1), the control input $u_i(t) \in \mathbb{R}^2$ is defined as $u_i \triangleq \begin{bmatrix} v_i(t) & \omega_i(t) \end{bmatrix}^T$, where $v_i(t), \omega_i(t) \in \mathbb{R}$ denotes the linear and angular velocity of robot i.

The workspace $\mathcal{F}$ is assumed to be circular and bounded with radius R_w, and $\partial\mathcal{F}$ denotes the boundary of $\mathcal{F}$. Each robot in $\mathcal{F}$ is assumed to have a limited communication and sensing capability encoded by a disk area with radius R_c and R_s respectively, and $R_c \geq R_s$[1]. For simplicity and without loss of generality, the following development is based on the assumption that the sensing area coincides with the communication area, and represented by radius R, i.e., $R_c = R_s = R$. Two moving robots can communicate and sense with each other if they stay within a distance of R. Further, it is assumed that all the robots have equal actuation capabilities.

One objective in this work is to lead the group of WMRs to rendezvous at a common destination p^* with a desired orientation θ_i^*, i.e., $q_i^* = \begin{bmatrix} (p^*)^T & \theta_i^* \end{bmatrix}^T$ for $\forall i$ in the workspace $\mathcal{F}$. The inter-robot communication of the WMRs is modeled as a *communication graph*, denoted as $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{V} = \{1, \cdots, N\}$ denotes the set of nodes, and $\mathcal{E}(t) = \{(i,j) \in \mathcal{V} \times \mathcal{V} | \, d_{ij} \leq R\}$ denotes the set of time varying edges, where node i and node j are located at a position p_i and p_j, and $d_{ij} \in \mathbb{R}^+$ is the relative distance defined as $d_{ij} = \|p_i - p_j\|$. In graph $\mathcal{G}(t)$, each node i represents a robot, and the edge (i,j) denotes a link between robot i and j when they stay within a distance of R. Each node i is assigned to a static subset $\mathcal{N}_i$, called the communication set, that includes the nodes with which it communicate. It is also assumed that the communication graph $\mathcal{G}$ is undirected, in the sense that $i \in \mathcal{N}_j \Longleftrightarrow j \in \mathcal{N}_i$ for $\forall i, j \in \mathcal{V}, i \neq j$. Due to the limited sensing and communication capabilities, robot i has only knowledge of the states (i.e., positions) of the robots within its sensing zone at each time instant, and exchange or share information with robots that belong to $\mathcal{N}_i$ through radio communication. Once robot j moves out of the sensing and communication zone of robot i, robot i will no longer obtain knowledge of the states of robot j directly. Hence, another objective is to maintain connectivity of the communication graph all the time.

The control objective is to derive a set of controllers to drive each robot to a desired rendezvous point with a desired orientation, while guaranteeing the communication graph remains connected during the system evolution, provided the given initial graph is connected. To achieve this goal, the subsequent development is based on the following assumptions.

Assumption 1: The initial graph $\mathcal{G}$ is connected, and those initial positions do not coincide with unstable equilibria (i.e., saddle points).

Assumption 2: The destination is achievable, which indicates that the destination will not meet any constraints, i.e., coincide with the workspace boundary, or lead to a disconnectivity of existing communication links.

III. Control Design

A. Dipolar Navigation Function

The navigation function is a particular category of potential function, whose negated gradient vector field is attractive toward the goal configuration and repulsive with respect to obstacles and the workspace boundary. Contrary to the artificial potential field-based approach, where the problem is the existence of local minima when attractive and repulsive force are combined, the navigation function is carefully

[1]The assumption of $R_c \geq R_s$ ensures that two robots are able to communicate with each other as long as they can sense each other.

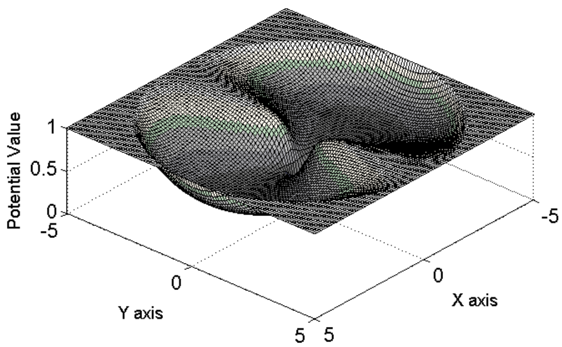

Fig. 1. An example of a dipolar navigation function with workspace of $R_w = 5$, and the destination located at the origin with a desired orientation $\theta^* = 0$.

designed such that the negated gradient field does not have any local minima and this closed-loop approach guarantees the convergence to a desired destination, as well as collision avoidance. Formally, a navigation function is defined as:

Definition 1: [12] [13], Let $\mathcal{F} \subset E^n$ be a compact connected analytic manifold with boundary. A map $\varphi : \mathcal{F} \to [0,1]$ is a Navigation Function, if it is: 1) smooth on $\mathcal{F}$ (at least a $\mathcal{C}^2$ function); 2) admissible on $\mathcal{F}$, (uniformly maximal on $\partial\mathcal{F}$ and constraint boundary); 3) polar on $\mathcal{F}$, (q_d is a unique minimum); and 4) a Morse function, (critical points of the navigation function are non-degenerate).

Specifically, property 2) establishes that the generated trajectories are collision-free, since the resulting vector field is transverse to the boundary of $\mathcal{F}$, while property 3) indicates that, using a polar function on a compact connected manifold with boundary, all initial conditions are either brought to a saddle point or to the unique minimum q_d. The last property ensures that the initial conditions that bring the system to saddle points are sets of measure zero [12]. Given this property, all initial conditions away from sets of measure zero are brought to the unique minimum.

Without taking into account the nonholonomic constraints, the original navigation functions, introduced in [12] and [13], are not suitable for the control of WMRs, since the feedback law generated from the gradient of the navigation function can lead to undesired behavior, like having the vehicle rotate in place. To overcome this undesired behavior, a Dipolar Navigation Function was developed in [25] and [14] with the advantage that the flow lines created in the potential filed resemble a dipole, so that the flow lines are all tangent to the desired orientation at the origin, and the vehicle is driven there with the desired orientation. One example of the dipolar navigation is shown in Fig. 1, where the potential field has a unique minimum at the destination (i.e., $p^* = [0,0]^T$ and $\theta^* = 0$), and achieves the maxima at the workspace boundary of $R_w = 5$. Note that the surface $x = 0$ divides the workspace into two parts, and forces all the flow lines to approach the destination parallel to the y-axis.

To navigate the robots and maintain network connectivity, the dipolar navigation function in [25] and [14] is modified as $\varphi : \mathcal{F} \to [0,1]$,

$$\varphi(\mathbf{P}) = \frac{\gamma}{(\gamma^\alpha + H_{nh} \cdot \beta)^{1/\alpha}}, \tag{2}$$

where $\alpha \in \mathbb{R}^+$ is a tuning parameter, and $\gamma : \mathbb{R}^2 \to \mathbb{R}^+$ is the goal function. The goal function γ in (2) encodes the control objective for the WMRs, specified by the distance from $\mathbf{P}(t) \in \mathbb{R}^{2N}$ to the common destination $\mathbf{P}^* \in \mathbb{R}^{2N}$, where $\mathbf{P}(t)$ denotes the stacked current position states, i.e., $\mathbf{P}(t) = \begin{bmatrix} p_1^T(t) & \cdots & p_N^T(t) \end{bmatrix}^T$, and $\mathbf{P}^*$ denotes the stacked destination. The goal function is designed as

$$\gamma(\mathbf{P}) = \|\mathbf{P} - \mathbf{P}^*\|^2. \tag{3}$$

The factor $H_{nh} \in \mathbb{R}$ in (2) creates the repulsive potential of an artificial obstacle, used to align the trajectories at the destination with the desired orientation. The repulsive potential factor is designed as

$$H_{nh} = \varepsilon_{nh} + \prod_{i=1}^{N} \eta_{nh_i}, \tag{4}$$

where ε_{nh} is a small positive constant, and $\eta_{nh_i} \in \mathbb{R}$ is designed as

$$\eta_{nh_i} = \left((\mathbf{P} - \mathbf{P}^*)^T \cdot n_{di}\right)^2,$$

where $n_{di} \in \mathbb{R}^{2N}$ is designed as

$$n_{di} = \begin{bmatrix} 0_{1\times 2(i-1)} & \cos(\theta_i^*) & \sin(\theta_i^*) & 0_{1\times 2(N-i)} \end{bmatrix}^T.$$

The constraint function $\beta : \mathbb{R}^2 \to [0,1]$ in (2) is designed as

$$\beta = \prod_{i=1}^{N} \beta_i = \prod_{i=1}^{N} \left(B_{i0} \cdot \prod_{j\in\mathcal{N}_i} b_{ij}\right), \tag{5}$$

to ensure communication links exist between robots within their sensing area and restrict the motion of each robot in the specified workspace during each time instant. Specifically, the constraint function in (5) is designed to vanish whenever node i intersects with one of the constraints in the environment, (i.e., if node i touches the workspace boundary, or departs away from its adjacent nodes $j \in \mathcal{N}_i$ to a distance of R_c). A collision region is defined for each agent i as a small disk area with radius $\delta_1 < R$ around the agent i. The function B_{i0} in (5) is used to model the potential collision of node i with the workspace boundary, where the positive scalar $B_{i0} \in \mathbb{R}$ is designed as

$$B_{i0} = \begin{cases} -\frac{1}{\delta_1^2} d_{i0}^2 + \frac{2}{\delta_1} d_{i0}, & d_{i0} < \delta_1 \\ 1, & d_{i0} \geq \delta_1, \end{cases} \tag{6}$$

where $d_{i0} \in \mathbb{R}$ is the relative distance of the node i to the workspace boundary defined as $d_{i0} = R_w - \|q_i\|$. To ensure the graph connectivity, an *escape region* for each agent i is defined as the outer ring of the communication area with radius r, $R - \delta_2 < r < R$, where $\delta_2 \in \mathbb{R}^+$ is a predetermined buffer distance. Edges formed with any node $j \in \mathcal{N}_i$ in the escape region are in danger of breaking. The control law for each robot i is designed to ensure the edges exist. In (5),

$b_{ij} \triangleq b(q_i, q_j) : \mathbb{R}^2 \to [0,1]$ ensures connectivity of the network graph (i.e., guarantees that nodes $j \in \mathcal{N}_i$ will never leave the communication zone of node i if node j is initially connected to node i) and is designed as

$$b_{ij} = \begin{cases} 1 & d_{ij} \le R - \delta_2 \\ -\frac{1}{\delta_2^2}(d_{ij} + 2\delta_2 - R)^2 + \frac{2}{\delta_2}(d_{ij} + 2\delta_2 - R) & R - \delta_2 < d_{ij} < R \\ 0 & d_{ij} \ge R. \end{cases} \tag{7}$$

Assumption 2 guarantees that γ and β will not be zero simultaneously. The navigation function candidate achieves its minimum of 0 when $\gamma = 0$ and achieves its maximum of 1 when $\beta = 0$. From the definition of navigation function, it is known that, if φ is a qualified navigation function, almost all initial positions (except for a set of measure zero points) asymptotically approach the desired destination. In on our previous work [3], the original navigation function modified to ensure connectivity, as designed in (7), is still a qualified navigation function. In [15], it is also shown that the navigation properties are not affected by the modification to a dipolar navigation with the design of (4), as long as the workspace is bounded, η_{nh_i} can be bounded in the workspace, and ε_{nh} is a small positive constant. As a result, the φ proposed in (2) can be proven to be a qualified navigation function. See [3] and [15] for further details.

B. Control Development

The desired orientation for robot i, denoted by $\theta_{di}(t)$, is defined as a function of the negated gradient of the navigation function (2) as follows

$$\theta_{di} \triangleq \arctan 2\left(-\frac{\partial \varphi}{\partial y_i}, \quad -\frac{\partial \varphi}{\partial x_i} \right), \tag{8}$$

where $\arctan 2(\cdot) : \mathbb{R}^2 \to \mathbb{R}$ denotes the four quadrant inverse tangent function, and $\theta_{di}(t)$ is confined to the region $\theta_{di}(t) \in (-\pi, \pi]$. By defining $\theta_{di}|_{p^*} = \arctan 2(0,0) = \theta_i|_{p^*}$, θ_{di} remains continuous along any approaching direction to the goal position. Based on the definition of θ_{di} in (8), the following expression can be obtained

$$\nabla_i \varphi = -\|\nabla_i \varphi\| \begin{bmatrix} \cos(\theta_{di}) & \sin(\theta_{di}) \end{bmatrix}^T, \tag{9}$$

where $\nabla_i \varphi$ denotes the partial derivative of φ with respect to p_i, $\nabla_i \varphi = \begin{bmatrix} \frac{\partial \varphi}{\partial x_i} & \frac{\partial \varphi}{\partial y_i} \end{bmatrix}^T$ and $\|\nabla_i \varphi\|$ denotes the Euclidean norm of $\nabla_i \varphi$. To quantify the navigation control objective, the difference between the current orientation configuration $\theta_i(t)$ and the desired orientation $\theta_{di}(t)$ for robot i at each time instant is defined as

$$\tilde{\theta}_i(t) = \theta_i(t) - \theta_{di}(t), \tag{10}$$

where the desired $\theta_{di}(t)$ is generated from the designed navigation function in (2) and (8). Since φ in (2) is a navigation function, the properties of a navigation function guarantees that $q_{di}(t) \to q_i^*$ as $t \to \infty$.

Based on the open-loop system introduced in (1) and the subsequent stability analysis, the controller for each robot is designed as

$$v_i = k_v \|\nabla_i \varphi\| \cos \tilde{\theta}_i, \tag{11}$$

$$\omega_i = -k_w \tilde{\theta}_i + \dot{\theta}_{di}, \tag{12}$$

where k_v, $k_w \in \mathbb{R}$ are positive, constant control gain. In (11) and (12), $\nabla_i \varphi$ and $\dot{\theta}_{di}$ are computed from (2) and (8) respectively as

$$\nabla_i \varphi = \frac{\alpha (H_{nh} \cdot \beta) \nabla_i \gamma - \gamma \nabla_i (H_{nh} \cdot \beta)}{\alpha(\gamma^\alpha + H_{nh} \cdot \beta)^{\frac{1}{\alpha}+1}}, \tag{13}$$

where $\nabla_i \gamma$ and $\nabla_i (H_{nh} \cdot \beta)$ are bounded in the workspace $\mathcal{F}$ from (3) and (5), and

$$\dot{\theta}_{di} = k_v \cos(\tilde{\theta}_i) \begin{bmatrix} \sin(\theta_{di}) \\ -\cos(\theta_{di}) \end{bmatrix}^T \nabla_i^2 \varphi \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}, \tag{14}$$

where$\nabla_i^2 \varphi$ denotes the Hessian matrix of φ with respect to p_i. Substituting (11) into (1), the closed-loop system can be obtained

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = k_v \|\nabla_i \varphi\| \cos \tilde{\theta}_i \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix}. \tag{15}$$

Taking the time derivative of $\tilde{\theta}_i(t)$ in (10) and using (1), the open-loop orientation tracking error system can be obtained as $\dot{\tilde{\theta}}_i = \omega_i - \dot{\theta}_{di}$. Using (12), the closed-loop orientation tracking error system is given by

$$\dot{\tilde{\theta}}_i = -k_w \tilde{\theta}_i, \tag{16}$$

and (16) can be solved as $\tilde{\theta}_i(t) = \tilde{\theta}_i(0) e^{-k_w t}$. From (13), (16) and (14), $\|\nabla_i \varphi\|$, $\tilde{\theta}_i$ and $\dot{\theta}_{di}$ is bounded within the workspace $\mathcal{F}$. Hence, in (11) and (12), the control efforts can be kept within the actuator constraints by choosing appropriate control gains, k_v and k_w.

IV. Connectivity and Convergence Analysis

A. Connectivity Analysis

Theorem 1: If the graph $\mathcal{G}$ is connected initially and $j \in \mathcal{N}_i$, driven by the desired orientation in (8), nodes i and j are ensured to be connected for all time, i.e., the set $\{p \,|\, \|p_i - p_j\| < R, \ j \in \mathcal{N}_i\}$ is invariant during the system evolution.

Proof: Consider node i located at a point $p_0 \in \mathcal{F}$ that causes $\prod_{j \in \mathcal{N}_i} b_{ij} = 0$, which will be true when either only one node j is about to disconnect from node i or when more than one node are about to disconnect with node i simultaneously. From (5), it indicates that $\beta_i = 0$, which results that $\beta = 0$. The navigation function designed in (2) achieves its maximum value whenever the constraints are met, i.e., $\beta = 0$. Thus, the navigation function φ is maximized at p_0. Since the desired orientation for each robot i in (8) is along the negated gradient of φ with respect to p_i, no open set of initial conditions can be attracted to the maxima of the navigation function driven by the desired orientation in (8).

Hence, it can be concluded node i will ensure connectivity with all nodes $j \in \mathcal{N}_i$ for all time, provided that node i moves according to the desired orientation in (8). ■

B. Convergence Analysis

Theorem 2: The controller developed in (11) and (12) along with the dipolar navigation function φ in (2) ensure the group of robots converge to the common point with the desired orientation, such as $\|p_i - p^*\| \to 0$ and $\left|\tilde{\theta}_i\right| \to 0$, for $\forall i \in \mathcal{N}$.

Proof: Consider a Lyapunov function candidate, $V = \varphi(\mathbf{P}(t))$. The time derivative of V is $\dot{V} = \nabla\varphi \cdot \dot{\mathbf{P}}$, where $\nabla\varphi \in \mathbb{R}^{2N}$ denotes the partial derivative of φ with respect to the stacked state vector $\mathbf{P}$. Using (15), the following expression can be obtained

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} \left(\begin{bmatrix} \frac{\partial\varphi}{\partial x_i}, & \frac{\partial\varphi}{\partial y_i} \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} \right) \qquad (17) \\ &= \sum_{i=1}^{N} k_v \|\nabla_i\varphi\| \cos\tilde{\theta}_i \left(\frac{\partial\varphi}{\partial x_i}\cos\theta_i + \frac{\partial\varphi}{\partial y_i}\sin\theta_i \right). \end{aligned}$$

Substituting (9) into (17) and using a trigonometric identity, $\dot{V}$ can be computed as

$$\dot{V} = \sum_{i=1}^{N} \left(-k_v \|\nabla_i\varphi\|^2 \cos^2\left(\tilde{\theta}_i\right) \right) \le 0.$$

Based on (3) and (7), it is clear that $\frac{\partial\varphi}{\partial x_i}, \frac{\partial\varphi}{\partial y_i} \in \mathcal{L}_\infty$ on workspace $\mathcal{F}$; hence (11) can be used to conclude that $v_i(t) \in \mathcal{L}_\infty$. Providing $\dot{\theta}_{di}(t) \in \mathcal{L}_\infty$ in (14) on $\mathcal{F}$, (12) can be used to show that $\omega_i(t) \in \mathcal{L}_\infty$. Applying LaSalle's invariance principle, the trajectories of the system converge to the largest invariant set contained in the set

$$S = \left\{ \|\nabla_i\varphi\| = 0 \text{ or } \cos^2\left(\tilde{\theta}_i\right) = 0,\ \forall i \in \mathcal{V} \right\}. \qquad (18)$$

Using the fact that $\tilde{\theta}_i(t) \to 0$ exponentially from (16), $\cos^2\left(\tilde{\theta}_i\right) \to 1$, hence, set S is reduced to the set

$$S' = \{\|\nabla_i\varphi\| = 0,\ \forall i \in \mathcal{V}\}. \qquad (19)$$

The set in (19) is formed whenever the potential functions either reach the destination or a saddle point. Since φ in (2) is a navigation function, it is shown that the saddle points of φ are isolated in [3]. Thus the set of initial conditions that lead to saddle points are sets of measure zero [26]. The largest invariant set constrained is the set of destination [27]. Hence, $\|\nabla_i\varphi\| = 0$ indicates that $\|p_i - p^*\| \to 0$ for $\forall i$. ■

V. Simulation

To illustrate the performance of the controller proposed in (11) and (12), a preliminary numerical simulation was performed to navigate a group of four mobile robots with the kinematics in (1) from an initially connected condition $\mathbf{q}(\mathbf{0})$, to the common destination $\left[(p^*)^T,\ \theta^*\right]^T$, which are specified as

$$\begin{bmatrix} q_1^T(0) \\ q_2^T(0) \\ q_3^T(0) \\ q_4^T(0) \end{bmatrix} = \begin{bmatrix} -2 & 1.5 & -1.4137 \\ -2 & 0.7 & -1.5708 \\ -2 & -0.7 & 1.7279 \\ -2 & -1.5 & 1.0996 \end{bmatrix},$$

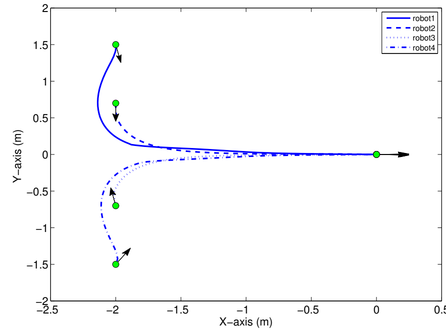

Fig. 2. Plot of the trajectory evolution for each mobile robot.

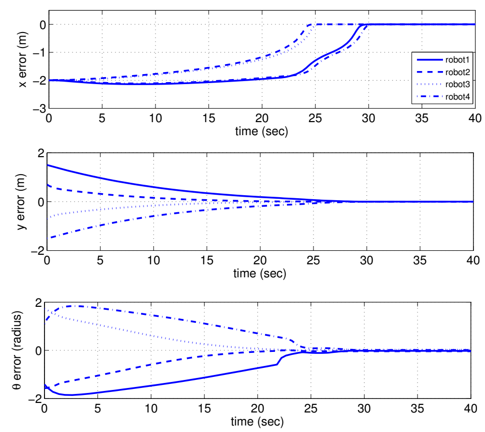

Fig. 3. Position and orientation error for each mobile robot.

and $\begin{bmatrix} (p^*)^T & \theta^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. The robots are distributed in a workspace of $R_w = 5\ m$. Each node is assumed to have a limited communication and sensing zone of $R = 2\ m$ and $\delta_1 = \delta_2 = 0.5\ m$. The control inputs designed in (11) and (12) were utilized to drive the group of mobile robots to the destination of p^* with the desired orientation θ^*. The tuning parameter α in (2) is $\alpha = 1.5$, while the control gains k_v and k_w are adjusted to $k_v = 0.9$ and $k_w = 1$.

The system is simulated for $40s$ and the simulation results are shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5. The actual trajectory evolution for each robot is shown in Fig. 2, where each robot is represented by a circle and the associated arrow indicates its current orientation. The resulting position and orientation errors for each mobile robot are depicted in the Fig. 3, which indicates that each robot achieves the common destination with desired orientation. The control inputs $v(t)$ and $\omega(t)$ for each robot is also shown in Fig. 4, which indicate a bounded control input. To show the communication links are always connected, the evolution of inter-robot distance is shown in Fig. 5. Since the inter-robot distance is less than the radius $R = 2\ m$ during the motion, connectivity is maintained.

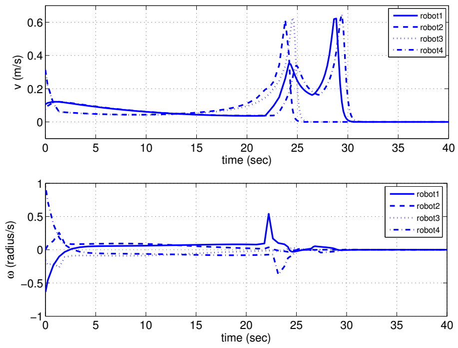

Fig. 4. Control actuation (i.e., linear velocity and angular velocity) for each mobile robot.

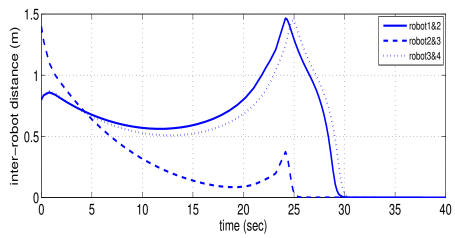

Fig. 5. The evolution of inter-robot distance.

VI. Conclusion

Based on the dipolar navigation function formalism, a time-varying continuous controller is developed for a multi-robot system to achieve a network cooperative goal, that is navigating the mobile robots to a common destination with a desired orientation, and ensuring the network is connected for all time, provided that the network is connected initially. Due to the limited communication and sensing capabilities, the robot is required to obtain the states of other robots through communication. Future research directions will include the development of a completely decentralized controller, which uses only local information within its sensing and communication zone, and results in radio silence during the motion.

References

[1] M. Zavlanos, A. Jadbabaie, and G. Pappas, "Flocking while preserving network connectivity," in *Proc. IEEE Conf. Decis. Control*, Dec. 2007, pp. 2919–2924.

[2] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sept. 2004.

[3] Z. Kan, A. Dani, J. M. Shea, and W. E. Dixon, "Ensuring network connectivity during formation control using a decentralized navigation function," in *Proc. IEEE Mil. Commun. Conf.*, San Jose, CA, 2010, pp. 954–959.

[4] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, June 2003.

[5] M. Ji and M. Egerstedt, "Connectedness preserving distributed coordination control over dynamic graphs," in *Proc. Am. Control Conf.*, 2005, pp. 93–98.

[6] D. Dimarogonas and K. Kyriakopoulos, "Connectivity preserving distributed swarm aggregation for multiple kinematic agents," in *Proc. IEEE Conf. Decis. Control*, Dec. 2007, pp. 2913–2918.

[7] D. Dimarogonas and K. Johansson, "Decentralized connectivity maintenance in mobile networks with bounded inputs," in *Proc. IEEE Int. Conf. Robot. Autom.*, May 2008, pp. 1507–1512.

[8] M. Zavlanos and G. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 812–816, Aug. 2007.

[9] ——, "Distributed connectivity control of mobile networks," *IEEE Trans. Robot.*, vol. 24, no. 6, pp. 1416–1428, Dec. 2008.

[10] A. Ghaffarkhah and Y. Mostofi, "Communication-aware target tracking using navigation functions - centralized case," in *Int. Conf. Robot Commun. Co-ord.*, March 31 - April 2 2009, pp. 1 –8.

[11] R. W. Brockett, R. S. Millman, and H. J. Sussmann, *Differential Geometric Control Theory*. Birkhauser, 1983.

[12] D. E. Koditschek and E. Rimon, "Robot navigation functions on manifolds with boundary," *Adv. Appl. Math.*, vol. 11, pp. 412–442, Dec 1990.

[13] E. Rimon and D. Koditschek, "Exact robot navigation using artificial potential functions," *IEEE Trans. Robot. Autom.*, vol. 8, no. 5, pp. 501–518, Oct 1992.

[14] H. Tanner and K. Kyriakopoulos, "Nonholonomic motion planning for mobile manipulators," in *IEEE International Conference onRobotics and Automation*, vol. 2, 2000, pp. 1233 –1238 vol.2.

[15] S. Loizou and K. Kyriakopoulos, "Navigation of multiple kinematically constrained robots," *IEEE Trans. Robot*, vol. 24, no. 1, pp. 221 –231, 2008.

[16] G. Roussos, D. Dimarogonas, and K. Kyriakopoulos, "3D navigation and collision avoidance for a non-holonomic vehicle," in *Proc. Am. Control Conf.*, 2008, pp. 3512 –3517.

[17] J. Chen, D. Dawson, M. Salah, and T. Burg, "Multiple UAV navigation with finite sensing zone," in *Proc. Am. Control Conf.*, June 2006, pp. 4933–4938.

[18] D. Dimarogonas, M. Zavlanos, S. Loizou, and K. Kyriakopoulos, "Decentralized motion control of multiple holonomic agents under input constraints," in *Proc. IEEE Conf. Decis. Control*, vol. 4, Dec. 2003, pp. 3390 – 3395.

[19] D. V. Dimarogonas, S. G. Loizou, K. J. Kyriakopoulos, and M. M. Zavlanos, "A feedback stabilization and collision avoidance scheme for multiple independent non-point agents,," *Automatica*, vol. 42, no. 2, pp. 229 – 243, 2006.

[20] D. Dimarogonas and K. Johansson, "Analysis of robot navigation schemes using Rantzer dual Lyapunov theorem," in *Proc. Am. Control Conf.*, June 2008, pp. 201 –206.

[21] H. Tanner and A. Kumar, "Towards decentralization of multi-robot navigation functions," in *Proc. IEEE Int. Conf. Robot. Autom.*, April 2005, pp. 4132 – 4137.

[22] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 863–868, May 2007.

[23] M. De Gennaro and A. Jadbabaie, "Formation control for a cooperative multi-agent system using decentralized navigation functions," in *Proc. Am. Control Conf.*, June 2006, pp. 1346–1351.

[24] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 916–922, May 2007.

[25] H. Tanner, S. Loizou, and K. Kyriakopoulos, "Nonholonomic navigation and control of cooperating mobile manipulators," *IEEE Trans. Robot. Autom.*, vol. 19, no. 1, pp. 53–64, Feb 2003.

[26] J. Milnor, *Morse theory*. Princeton Univ Pr, 1963.

[27] D. Dimarogonas and K. Kyriakopoulos, "A feedback control scheme for multiple independent dynamic non-point agents," *Int. J. Control*, vol. 79, no. 12, pp. 1613–1623, 2006.